Asian Journal of Statistical Sciences
Vol. 4, No-2, (2024), pp. 83-101

ARF INDIA
Academic Open Access Publishing
Journal homepage link:
http://arfjournals.com/ajss

A New Structure and Modus Operandi for Reliability Attribute Acceptance Sampling Plans using bivariate Poisson distribution with its application in One-shot Syringes

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ARTICLE HISTORY
Compiled April 2, 2025

Received 05 July 2024; Accepted 20 December 2024

ABSTRACT
Acceptance sampling plans are essential in industries like manufacturing as they aid in making decisions on whether to accept or reject a lot, depending on samples of the population. The reliability single sampling plan is typically used for estimating time characteristics and optimizing sample size when reliability is a critical quality factor. This article implements a reliability sampling plan using the implementation of bivariate Poisson distribution and lifetime distribution. In appropriate circumstances, the bivariate Poisson distribution may be utilized as a basis for identifying items with defects, and lifetime distributions will be used to determine the defectiveness. In the case of consumer risk, the minimum sample size is needed to attain the specified life percentile. An acceptable lot's expected percentile time and operating characteristic values are given. Thus, this paper is concerned with developing a new structure and modus operandi for estimating percentiles using bivariate Poisson distribution with its application in one-shot syringes data quality management.

KEYWORDS
Bivariate Poisson distribution; minimum sample size; operating characteristic function; producers risk; consumers risk; percentile time.

1. Introduction

Acceptance sampling plan (ASP) was developed by Dodge and Romig in the early 1930s and is now an essential part of statistical quality control. A random sample should be selected from the lot, and the lot's disposal should be decided based upon the information produced from the sample inspection. In general, the decision is whether to accept or reject the lot. This procedure is referred to as Lot Acceptance Sampling or simply Acceptance Sampling, and it is frequently utilized in industrial quality control throughout the manufacturing process. Graves et al. (2000) illustrated how acceptance sampling can improve system reliability. Sampling inspection counts the number of failures in a random sample of the population to assess if the population is approved or rejected. Montgomery (2013) stated that the variable sampling makes use of ongoing measurements of a system's quality parameters, including its mass, length, or failure

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time. As part of attribute sampling, components must be categorized as either defective or non-defective. Variables are associated with the measurement mean, whereas characteristics are relevant to the fraction of faulty systems. An acceptance reliability sampling plan determines whether to reject or accept the appropriate lot based on the lifetime of the items found on tests as well as the number of failures identified within a pre-specified testing time, taking into consideration for both the producers and the consumers risk. Epstein (1954), Sobel and Tischendorf (1959), Goode and Kao (1961), Gupta (1962), Fertig and Mann (1980), Kantam and Rosaiah (1998), Kantam et al. (2001), Baklizi (2003), Wu and Tsai (2005), Rosaiah and Kantam (2005), and Tsai and Wu (2006) developed acceptance sampling plans based on truncated life tests.

A common design for ASP is one that uses the population mean under truncated life tests. These ASPs might not, however, meet the engineering requirements of the particular strength percentile. Regarding the functionality of a disclosed lower percentile, many items with a lower percentile that falls short of customer criteria may be allowed by the ASP on the basis of overall mean. Moreover, small percentiles of interest may change significantly downward in response to a slight fall in the mean and a concurrent slight rise in variability. This implies that many goods could be accepted with only a slight drop on an average life after the evaluation. However, the products material strengths have greatly declined and might not live up to consumer expectations. In life testing applications, engineers therefore focus more on the lifetime percentiles compared to average life. For truncated life tests, acceptance sampling using percentiles was established by Balakrishnan et al. (2007), Lio et al. (2009), Rao and Kantam (2010), Srinivasa Rao and Kantam (2013), Rao and Naidu (2014), Pradeepa Veerakumari and Ponneeswari (2016, 2017), and Kumarasamy and Thottathil (2022). Percentiles are used to show additional details regarding the life distribution as the average life. Through the use of ERD percentiles, the Reliability Acceptance Single Sampling Plan (RASSP) describes the application of bivariate Poisson distribution (BPD) in ASPs. The operating characteristic (OC) curves, some pertinent tables, and the suggested sampling plans are provided in the section Illustrative Example for the ERD with a truncated life test. The conclusion section provides the key findings and insights.

2. Significance of Acceptance Sampling Plan procedures in One-Shot devices

Acceptance sampling is a statistical technique for one-shot devices that estimates a population attribute from a sample in order to forecast the probability of success or dependability. A one-shot device is defined as equipment, and it can be used once in its lifetime. Such devices are retained in maintenance until they are inspected for reliability and are also destructive in nature at the time of inspection. When devices are kept in storage for an extended period of time, they eventually fail. Sampling inspection is very appropriate for preventing device destruction since destructive testing makes the manufacturing costs of the devices wastable, and it is not desirable to do a thorough inspection. One-shot devices are unknown in their operational condition until they are tested. The device becomes useless after usage. One can only determine whether the failure time occurs before or after the inspection time because the one-shot device is destroyed after usage.

Many things that we use every day are classified as non-repairable, including syringes, satellites, electrical lights, weapons, and non-biodegradable batteries. These products are sometimes referred to as instantaneous-duty goods or one-shot devices. They might be referred to as use and throw devices. The Coalition for Safe Community Needle Disposal collected data indicating that the total amount of syringes utilized by households each year is estimated to be more than 7.5 billion, and it is increasing. The automated inspection device serves to examine the product regarding particulates and cosmetic defects, including accurate plunger placement. Balakrishnan et al. (2021), Zhao and Yun (2020), Vo et al. (2016), and Challener (2019) developed articles related to one-shot devices and also one-shot syringes.

3. Glossary of notations

n - Sample size
c_1 - Acceptance number for the inspection based on examining the quality when there are visible flaws when opening the syringe
c_2 - Acceptance number for the inspection based on examining the quality when the injection force of the syringe is high
d_1, d_2 - Number of defectives based on examining the two quality characteristics
p - Proportion defective
t_q - Actual q^{th} percentile
t_q^0 - Specified q^{th} percentile
δ - $\frac{t}{t_q}$
δ_0 - $\frac{t}{t_q^0}$

4. Implication of Reliability Acceptance Single Sampling Plan through percentiles of ERD (RASSP)

An inspection process called ASSP helps make decisions on whether or not to accept or just reject a particular lot. The parameter is considered to follow a bivariate Poisson distribution with a parameter since larger lots are taken and success and failure are experienced frequently. The following assumptions apply to the development of ASP using ERD percentiles:

(1) The proposed single sample plan approach is assumed to follow a bivariate Poisson distribution.
(2) The failure probability observed during a particular time t is denoted by $p = F(t; \delta_0)$
(3) Assume that the acceptance number is c_1, c_2. We accept an entire lot if, at the appropriate time, there are fewer failures than c.

$$F(t; \delta) \leq F(t; \delta_0) \Leftrightarrow t_q \geq t_q^0 \tag{1}$$

4.1. Bivariate Poisson distribution in Framework

Campbell (1934) and Aitken (1936) were the first to discuss the BPD. M'Kendrick (1926) is credited as the most remarkable evolution of the BPD, having predicted many contemporary concepts. It was revived by Irwin (1963), who also provided a very good discussion. The most used model for bivariate counts is the bivariate Poisson.

Holgate (1964) proposed it, and Johanson Fand Kotz (1969) delivered it. The primary source of the BPD found in the literature is Campbell (1934). The work of Holgate (1964) and Teicher (1954), which is fundamentally based on the work of Campbell (1934), appears to be the only one that takes into consideration difficult examples of multivariate Poisson distributions. The BPD can be produced as a limiting form of the bivariate binomial distribution, as demonstrated by Hamdan and Al-Bayyati (1969). Lakshminarayana et al. (1999) developed the bivariate Poisson distribution and studied its distributional characteristics. Campbell (1934) defines the distribution as the limit of a bivariate binomial distribution using a probability generating function (PGF).

$$\pi(t_1, t_2) = [1 + p_{1+}(t_1 - 1) + p_{+1}(t_2 - 1) + p_{11}(t_1 - 1)(t_2 - 1)]^n \tag{2}$$

Now let λ_1, λ_2, and λ_1 be positive constants with respect to n. Put

$p_{1+} = \frac{\lambda_1}{n}$, $p_{+1} = \frac{\lambda_2}{n}$, $p_{11} = \frac{\lambda_{11}}{n}$

and

$$\pi(t_1, t_2) = [1 + \frac{\lambda_1}{n}(t_1 - 1) + \frac{\lambda_2}{n}(t_2 - 1) + \frac{\lambda_{11}}{n}(t_1 - 1)(t_2 - 1)]^n \tag{3}$$

Applying limits as $n \Rightarrow \infty$, we have

$$\pi(t_1, t_2) = exp(\lambda_1(t_1 - 1) + \lambda_2(t_2 - 1) + \lambda_{11}(t_1 - 1)(t_2 - 1)) \tag{4}$$

With the parameters λ_1, λ_2, and λ_{11}, the random variables (X, Y) are considered to have BPD. Assume that W_1, W_2, and W_3 are independent Poisson random variables with λ_1, λ_2, and λ_3 as their respective parameters. Assume that $X = W_1 + W_2$ and $Y = W_2 + W_3$. The combination of random variables (X, Y) having probability function $f(x, y)$ possesses an $E[t_1^X t_2^Y]$ PGF. We will represent PGF by $\pi(t_1, t_2)$ and which can be expressed as

$$\pi(t_1, t_2) = exp(\lambda_1(t_1 - 1) + \lambda_2(t_2 - 1) + \lambda_3(t_1 t_2 - 1)) \tag{5}$$

which is derived from equation (4). The marginal generating functions are easily obtained from the joint PGF (5).

$$\pi_i(t) = exp[(\lambda_1 + \lambda_3)(t - 1)], i = 1, 2 \tag{6}$$

The one way of writing the probability mass function (PMF) of BPD is provided by Holgate (1964) in (6) as follows:

$$f(x, y) = \frac{e^{-\lambda_1}\lambda_1^x}{x!}\frac{e^{-\lambda_2}\lambda_2^y}{y!}e^{-\lambda_3}\sum_{i=0}^{\min(x,y)}\binom{x}{i}\binom{y}{i}i!\left\{\frac{\lambda_3}{\lambda_1\lambda_2}\right\}^i \tag{7}$$

The trivariate reduction approach determines the probability of the occurrence $\{X = r, Y = s\}$. This occurrence is similar to the union of mutually exclusive events $\{W_1 = r - i, W_2 = s - i, W_3 = i\}$ for $i = 0, 1, \ldots, \min(r, s)$. The bivariate distribution accommodates the dependency of two random variables. Each of the random variables follows a Poisson distribution, whereby

$$E(X) = V(X) = \lambda_1 + \lambda_3$$

and

$$E(X) = V(X) = \lambda_1 + \lambda_3$$

Furthermore, $cov(X, Y) = \lambda_3$, indicating a measure of dependency within the variables that are selected at random. If $\lambda_3 = 0$, then both variables are actually independent, and the BPD is the same as their product, which has two independent Poisson distributions.

$$f(x, y) = \frac{e^{-\lambda_1}\lambda_1^x}{x!}\frac{e^{-\lambda_2}\lambda_2^y}{y!} \tag{8}$$

Demonstrating that in this instance, $X \sim P(\lambda_1)$ and $Y \sim P(\lambda_2)$, independent of one another.

4.2. Exponentiated Rayleigh Distribution

In response to an issue in the field of acoustics, Rayleigh (1880) first developed the Rayleigh distribution, which has been utilized as a lifetime distribution in reliability for a number of years. A specific instance of the Weibull distribution, the Rayleigh distribution finds extensive use in a variety of fields, including electro-vacuum device life testing Polovko (1968) and communication engineering Dyer and Whisenand (1973a, 1973b). The Rayleigh distribution's probability density function (PDF) and cumulative distribution function (CDF) are given by

$$f(t : \tau) = \frac{t}{\tau^2}e^{-\frac{1}{2}\left(\frac{t}{\tau}\right)^2}; t > 0, \tau > 0 \tag{9}$$

and

$$F(t, \tau) = 1 - e^{-\frac{1}{2}(\frac{t}{\tau})^2}, t > 0, \tau > 0 \tag{10}$$

The failure rate of the Rayleigh distribution is a crucial feature that increases linearly with time. Because of this characteristic, it's a good model to use for parts that might not have any manufacturing flaws yet age quickly.

A model known as the exponentiated distribution was presented by Gupta (1998) for analyzing time of failure using $F^*(t) = [F(t)]^\theta$, where θ represents the positive real value generated from Lehman alternatives and $F(t)$ is the baseline distribution function. According to Abdullah (2015), a CDF F(.) that has a parameter α (a positive real integer) added to it through exponentiation yields a CDF known as the Exponentiated distribution (ED). The CDF of the exponentiated distribution is expressed below.

$$G(x) = G(X; \theta) = [F(X; \beta)]^\alpha \equiv [F(X)]^\alpha \tag{11}$$

An alternative estimator for the generalized Rayleigh distribution was derived by Kundu and Raqab (2005). Additionally, the Rayleigh Distribution's distribution function is provided by

$$F(t,\tau) = 1 - e^{-\frac{1}{2}(\frac{t}{\tau})^2}, t > 0, \frac{1}{\tau} > 0 \tag{12}$$

Consequently, the ERDs CDF is provided by

$$F(t,\tau,\theta) = [1 - e^{-\frac{1}{2}(\frac{t}{\tau})^2}]^{\theta}, t > 0, \frac{1}{\tau} > 0, \theta > 0 \tag{13}$$

where the scale parameter is represented by τ and the shape parameter is represented by θ. Thus, the ERD's PDF could be expressed with

$$f(t,\tau,\theta) = \frac{d}{dt}\left[F(t,\tau,\theta)\right] = \frac{d}{dt}\left[1 - e^{-\frac{1}{2}(\frac{t}{\tau})^2}\right]^{\theta}, t > 0, \tau > 0, \theta > 0 \tag{14}$$

$$f(t,\tau,\theta) = \theta\left[1 - e^{-\frac{1}{2}(\frac{t}{\tau})^2}\right]^{\theta-1}\left[\frac{t}{\tau^2}e^{-\frac{1}{2}(\frac{t}{\tau})^2}\right], t > 0, \tau > 0, \theta > 0 \tag{15}$$

The probability of failure within short periods of time is known as the hazard function. The hazard function shows an instantaneous failure rate at time t, assuming that the product never fails prior to t. The ratio of the PDF to the survival function, which is sometimes referred to as the hazard rate, failure rate, or force of mortality. And it is presented by,

$$h(t) = \frac{f(t)}{1 - F(t)} \tag{16}$$

Thus, for ERD, the hazard function is

$$h(t) = \frac{\theta[1 - e^{-\frac{1}{2}(\frac{t}{\tau})^2}]^{\theta-1}[\frac{t}{\tau^2}e^{-\frac{1}{2}(\frac{t}{\tau})^2}]}{1 - [1 - e^{-\frac{1}{2}(\frac{t}{\tau})^2}]^{\theta}} \tag{17}$$

4.3. Assumptions

We consider the following assumption:

There are N separate one-shot syringes in storage. The duration of storage of the syringe is distributed according to a bivariate Poisson model. Failures of one-shot devices can only be found by destructive inspection on a regular basis. Out of N systems, a random selection is made for inspection. Every inspection has a set sample size. The time needed for replacement and inspection is insignificant.

4.4. Singe Sampling Plan

(1) Choose a randomized sample with size of n units in the lot and examine each of the units for conformance with the provided attribute requirements.
(2) Count the number of defectives d_1 in the inspection when there are visible flaws when opening the syringe and count the number of defectives d_2 in the inspection when the injection force of the syringe is high.
(3) If $d_1 \leq c_1$ and $d_2 \leq c_2$, accept all of the lot; if not, reject it.

4.5. Percentile Estimator

The term percentile is used to express the way a score relates to various scores within a similar set. A set of sorted data is divided into hundreds by percentiles. Any distributions percentile can be determined by

$$P(T \leq t_q) = q \tag{18}$$

$$t_q = \tau\sqrt{-2ln(1 - q^{\frac{1}{\theta}})} \tag{19}$$

Let

$$\eta = \sqrt{-2ln(1 - q^{\frac{1}{\theta}})} \tag{20}$$

$$\tau = \frac{t_q}{\eta}$$

The CDF of the ERD is given by equation (13) when the scale parameter (τ) is substituted.

$$F(t) = \left[1 - e^{-\frac{1}{2\eta^2}(\frac{t}{t_q})^2}\right]^{\theta}; t > 0, \theta > 0 \tag{21}$$

Letting $\delta = \frac{t}{t_q}$

$$F(t;\tau,\theta) = [1 - e^{-\frac{1}{2}(\eta\delta)^2}]^{\theta} \tag{22}$$

Considering the partial derivative in terms of δ provides

$$\frac{\partial F(t;\delta)}{\partial \delta} = \theta\eta[1 - e^{-\frac{1}{2}(\eta\delta)^2}]^{\theta-1}[e^{-\frac{1}{2}(\eta\delta)^2}]$$

Since $\frac{\partial F(t;\delta)}{\partial \delta} > 0, F(t;\delta)$ is an increasing function of δ.

4.6. Minimum Sample Size

The proposed sampling plan is defined by $(n, c_1, c_2, t/t_q^0)$ for a specified P^*. In this case, the lots are large enough to allow for the use of the BPD. The task is to find the least positive number, n, necessary to assert that $t_q > t_q^0$ for the given values of $P^*(0 < P^* < 1), t_q^0$ and c_1, c_2.

$$\sum_{i=0}^{c_1} \frac{e^{-np}np^i}{i!} . \sum_{i=0}^{c_2} \frac{e^{-np}np^i}{i!} \leq 1 - P^* \tag{23}$$

$p = F(t; \delta_0)$, which solely depends on $\delta_0 = tt_q^0$, is the probability of failure at time t given the specified q^{th} percentile lifetime t_q^0. As $\partial F(t, \delta)/\partial\delta > 0, F(t; \delta)$ is a function of δ that does not decrease. Since $\partial F(t, \delta)/\partial\delta > 0$. Consequently, we have

$$F(t; \delta) \leq F(t; \delta_0) \Leftrightarrow \delta \geq \delta_0 \tag{24}$$

Any given q, tt_q^0, P^* can provide the smallest sample n meeting the inequalities. Tables 1 and 4 show only the results of small sample size for q=0.1, tt_q^0= 0.7,0.9,1.0,1.5,2.0,2.5,3.0,3.5, P^*= 0.75,0.90,0.95,0.99, c_1, c_2 = 0,1,2,3,4,5,6,7,8,9,10.

4.7. Operating Characteristic Function

The acceptance sampling strategy can be described, and its flaws can be found with the help of the OC function. Following plan preparation, the OC function serves as an effective arbiter for the manufacturer and the client, evaluating performance and contrasting it with other sampling plans. The possibility of a lot can be accepted with a specific acceptance sampling strategy if the batch or process meets a specific quality standard, known as a probability of acceptance, or $P_a(p)$. The curve of OC evaluates $P_a(p)$ with possible amounts for the proportion defective.The OC function is in terms of a single sampling plan of inspection based on examining the quality when there are visible flaws when opening the syringe.

$$P_{a_1}(p) = P(d_1 \leq c_1) = \sum_{i=0}^{c_1} \frac{e^{-np}np^i}{i!} \tag{25}$$

The OC function is in terms of a single sampling plan of inspection based on examining the quality when the injection force of the syringe is high.

$$P_{a_2}(p) = P(d_2 \leq c_2) = \sum_{i=0}^{c_2} \frac{e^{-np}np^i}{i!} \tag{26}$$

The sampling plan's operating characteristic function provides the probability of accepting the lot $P_a(p)$ based on the BPD, which is given by

$$P_a(p) = P_{a_1}(p) * P_{a_2}(p)$$

$$P_a(p) = \sum_{i=0}^{c_1} \frac{e^{-np} np^i}{i!} . \sum_{i=0}^{c_2} \frac{e^{-np} np^i}{i!} \tag{27}$$

Equation (24) can be used to calculate the acceptance probability for the values of n,c_1, c_2, and p. During the truncated life test, the ASP for percentiles must meet the minimal sample size n for the provided acceptance numbers c_1 and c_2 to ensure consumer risk does not exceed more than $1 - P^*$. A bad lot occurs when the specified percentile, t_q^0, is greater than the true 100 q^{th} percentile, t_q. Consequently, the probability P^* represents a confidence level since it is at least equal to the possibility of rejecting a bad lot with $t_q > t_q^0$. Thus, the proposed ASP for a given P^* can be represented by $(n, c_1, c_2, t/t_q^0)$.

4.8. Producers Risk

The likelihood of rejecting a lot when $t_q > t_q^0$ is known as the producers risk. The purpose is to know the value of d_q for a certain producer risk, lets say, such that if a sampling plan (n, c_1, c_2, tt_q^0) is established at a specific confidence level P^*, we can be sure the producers risk is less than or equal to α. As a result, in considering (27), the least number d_q must be determined.

$$\sum_{i=0}^{c_1} \frac{e^{-np} np^i}{i!} . \sum_{i=0}^{c_2} \frac{e^{-np} np^i}{i!} \geq 1 - \alpha \tag{28}$$

where $p = F(\frac{t}{t_q^0} \frac{1}{d_q}), d_q = \frac{t_q}{t_q^0}$.

4.9. Design of the table

(1) Determine η's value when θ=2 and then q=0.1.
(2) Fix the evaluated $\eta, c_1 = c_2 = 2$ and $t/t_q = 0.7, 0.9, 1, 1.5, 2, 2.5, 3, 3.5$ and 4.
(3) Determine the minimum value for n that satisfies $\sum_{i=0}^{c_1} \frac{e^{-np} np^i}{i!} . \sum_{i=0}^{c_2} \frac{e^{-np} np^i}{i!} \leq 1 - P^*$, where P^* represents the probability of rejecting a bad lot.
(4) Determine a ratio of $d_{0.1}$ for the obtained n value so that $\sum_{i=0}^{c_1} \frac{e^{-np} np^i}{i!} . \sum_{i=0}^{c_2} \frac{e^{-np} np^i}{i!} \geq 1 - \alpha$, where, α=0.05, $p = F(\frac{t}{t_q^0} \frac{1}{d_q}), d_q = \frac{t_q}{t_q^0}$.

5. Illustrative Example

The primary attributes of one-shot syringe inspection are as follows: It's not unexpected that there are several chances for these intricate systems to fall short of quality standards, considering the variety of tests that need to be performed on both empty and loaded syringes.

- Failures with filled syringes are contingent upon the design of a medicinal component.
- Some mistakes involve problems pertaining to the patient. It is possible for patients to experience difficulties with the combined product (consumers handling),

and these problems ought to be regarded as testing failures. Examples include excessive injection forces, lengthy injection durations, and general syringe gripping problems.

- For pre-filled syringes, tests are also conducted on the delivered and retention volumes. Since it has an impact on filling capacity along with packing tolerance throughout manufacturing, the retained volume is significant.
- Applying insufficient silicone oil to the syringe's barrel is a major reason why functional tests fail. Inadequate oil application can cause the plunger to clatter or stop moving through the barrel and can also make it impossible to get the plunger moving at all. As a result, this concludes using a reduced inspection plan with a small sample population, thereby increasing the customer's beta risk.

In the acceptance sampling plan, we check two different quality characteristics: $Q_1 =$ examining the quality when there are visible flaws. $Q_2 =$ examining the quality when the injection force of the syringe is high; the two attributes of quality are independent. The acceptance probability of the sample with size $n = 11$ and acceptance number $c_1 = 2$ is based on the visible flaws and the acceptance probability of the sample with size $n = 11$ and acceptance number $c_2 = 2$ is based on the high injection force. Considering the ERD of the lifetime distribution, the ratio $tt_q^0 = 1.5$ should have resulted from the experimenters aim to ascertain the actual unidentified 10^{th} percentile lifetime value for one-shot syringe storage, which was intended to be 2 years, and the termination of the life test at 3 years. Therefore, the necessary sample size n, which can be obtained from Table 1, should be at least 11 for an acceptance number of $c_1 = 2$ and $c_2 = 2$ and the confidence level $P^*= 0.90$. As a result, in this instance, $(n, c_1, c_2, tt_q^0) = (11,2,2,1.5)$ should be the ASP from truncated life tests for the ERD 10^{th} percentile. From Table 3, the OC values under ERD for the acceptance sampling plan $(n, c_1, c_2, tt_q^0) = (11,2,2,1.5)$ and confidence level P^*=0.90 are as follows:

$t_{0.1}/t_{0.1}^0$	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
$L(p)$	0.0013	0.0880	0.4746	0.8108	0.9434	0.9834	0.9949	0.9983	0.9994

This demonstrates that when the true and specified 10^{th} percentiles are the same ($t_{0.1}/t_{0.1}^0 = 1$), the producers risk approximates 0.912(= 1-0.0880). If the actual 10^{th} percentile exceeds or equals 2.75 times the specified 10^{th} percentile, the producers risk approaches zero.

Table 2 provides experimenters with the true 10^{th} percentile estimate with the level of confidence 95% on various acceptance numbers and $t/t_{0.1}^0$ values in accepted lots. In the above case, $d_{0.1}$ should be 1.7733 with $c_1, c_2 = 2$, $t/t_{0.1} = 1.5$ and $P^* = 0.90$. This implies that the products 10^{th} percentile life is required to be 1.7733 times greater than the minimum required lifetime in order to be accepted under the aforementioned ASP with a probability of at least 0.95. As an alternative, lets consider that consumers wish to reject a bad lot at a probability of $P^*= 0.95$ as well as the products possessing an ERD. So, whatever the actual 10^{th} percentile life for these products remains, the producer's risk is 0.05 when the ASP depends upon an acceptance number $c_1, c_2 = 2$ and $t/t_{0.1}$=1. Table 2 shows that the entry for $P^* = 0.95$, $c_1, c_2 = 2$ and $t/t_{0.1}$=1 is $d_{0.1}$= 1.7099. Consequently, its essential that the manufacturers product be accepted with a probability of 0.95 with the aforementioned ASP, it must have a 10^{th} percentile life that is at least 1.7099 times the specified 10^{th} percentile. According to Table 1, the acceptance sampling plan $(n, c_1, c_2, tt_q^0) = (42, 2, 2, 1)$ requires $n = 42$ products to be tested.

Similarly, the necessary sample size n, which can be obtained from Table 4, should be at least 9 for an acceptance number of $c_1 = 1$ and $c_2 = 2$ and the confidence level $P^*=$ 0.90. As a result, in this instance, $(n, c_1, c_2, tt_q^0) =$ (9,1,2,1.5) should be the ASP from truncated life tests for the ERD 10^{th} percentile. From Table 4, the OC values under ERD for the acceptance sampling plan $(n, c_1, c_2, tt_q^0) =$ (9,1,2,1.5) and confidence level $P^* = 0.90$ are as follows:

$t_{0.1}/t_{0.1}^0$	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
$L(p)$	0.0023	0.0871	0.4115	0.7239	0.8838	0.9507	0.9781	0.9897	0.9949

This demonstrates that when true and specified 10^{th} percentiles are the same ($t_{0.1}/t_{0.1}^0 = 1$), producers risk approximates 0.9129(= 1-0.0871). If the actual 10^{th} percentile exceeds or equals to 2.75 times the specified 10^{th} percentile, the producers risk approaches zero. Table 2 provides experimenters with the true 10^{th} percentile estimate with the level of confidence 95% on various acceptance number and $t/t_{0.1}^0$ values in accepted lots. In the above case, $d_{0.1}$ should be 1.9933 with $c_1 = 1, c_2 = 2, t/t_{0.1} = 1.5$ and $P^* = 0.90$. This implies that the products 10^{th} percentile life is required to be 1.9933 times greater than the minimum required lifetime in order to be accepted under the aforementioned ASP with a probability of at least 0.95. As an alternative, lets consider that consumers wish to reject a bad lot at a probability of $P^* = 0.95$ as well as the products possess an ERD. So, whatever the actual 10th percentile life for these products remain thus the producer's risk is 0.05 when the ASP depends upon an acceptance number $c_1 = 1, c_2 = 2$ and $t/t_{0.1} = 1$. The Table 5 shows that the entry for $P^* = 0.95$, $c_1 = 1, c_2 = 2$ and $t/t_{0.1} = 1$ is $d_{0.1} = 1.9023$. Consequently, its essential that manufacturers product be accepted with a probability of 0.95 with the aforementioned ASP, it must have a 10^{th} percentile life that is at least 1.9023 times the specified 10^{th} percentile. According to Table 4, the acceptance sampling plan $(n, c_1, c_2, tt_q^0) =$ (34,1,2,1) requires $n = 34$ products to be tested.

From Table 7, the OC values under ERD for the acceptance sampling plan $(n, c_1, c_2, tt_q^0) =$ (13,2,3,1.5) and confidence level P^*=0.90 are as follows:

$t_{0.1}/t_{0.1}^0$	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
$L(p)$	0.0006	0.0748	0.4731	0.8202	0.9488	0.9854	0.9956	0.9986	0.9995

From Table 8, the OC values under ERD for the acceptance sampling plan $(n, c_1, c_2, tt_q^0) =$ (16,3,4,1.5) and confidence level $P^* = 0.90$ are as follows:

$t_{0.1}/t_{0.1}^0$	0.75	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
$L(p)$	0.0004	0.0889	0.5738	0.8994	0.9810	0.9964	0.9993	0.9998	1.000

Table 1. Minimum sample sizes required to prove that the 10^{th} percentile exceeds the values provided, $t^0_{0.1}$, with the probability P^* and same acceptance numbers c_1, c_2 using the bivariate Poisson approximation.

P^*	c_1, c_2	$t/t_{0.1}$								
		0.7	0.9	1	1.5	2	2.5	3	3.5	4
0.75	1	59	24	17	6	3	3	2	2	2
0.75	2	93	39	27	9	5	4	3	3	3
0.75	3	128	53	37	12	7	5	4	4	4
0.75	4	162	67	47	15	8	6	5	5	5
0.75	5	197	81	57	18	10	7	7	6	6
0.75	6	231	95	67	21	11	9	8	7	7
0.75	7	266	110	77	24	13	10	8	8	8
0.75	8	301	124	87	27	15	11	10	9	9
0.75	9	335	138	97	30	16	12	11	10	10
0.75	10	370	152	107	33	18	13	12	11	11
0.9	1	82	34	24	8	4	3	3	3	3
0.9	2	123	51	36	11	6	5	4	4	4
0.9	3	162	67	47	15	8	6	5	5	5
0.9	4	200	83	58	18	10	8	7	6	6
0.9	5	239	98	69	21	12	9	8	8	7
0.9	6	277	112	80	25	14	10	9	9	9
0.9	7	314	130	91	28	15	12	10	10	10
0.9	8	352	145	102	31	17	13	11	11	11
0.9	9	389	160	113	34	19	14	12	12	12
0.9	10	427	176	124	38	21	15	14	13	13
0.95	1	99	41	29	9	5	4	4	3	3
0.95	2	143	59	42	13	7	5	5	5	5
0.95	3	185	76	54	17	9	7	6	6	6
0.95	4	226	93	66	20	11	8	7	7	7
0.95	5	266	110	77	24	13	10	9	8	8
0.95	6	306	128	89	27	15	11	10	9	9
0.95	7	345	142	100	31	17	13	11	11	11
0.95	8	385	158	111	34	19	14	12	12	12
0.95	9	424	175	123	37	21	15	14	13	13
0.95	10	462	190	134	41	22	17	15	14	14
0.99	1	135	56	39	12	7	5	5	4	4
0.99	2	185	76	54	17	9	7	6	6	6
0.99	3	232	96	67	21	11	9	8	7	7
0.99	4	277	114	80	25	14	10	9	9	9
0.99	5	322	133	93	29	16	12	10	10	10
0.99	6	365	150	106	32	18	13	12	11	11
0.99	7	407	168	118	36	20	15	13	12	12
0.99	8	450	186	130	40	22	16	14	14	14
0.99	9	492	203	143	43	24	18	16	15	15
0.99	10	534	220	155	47	26	19	17	16	16

Table 2. Lot acceptance ratio $d_{0.1}$ for the ERD with a 0.05 producer's risk.

P^*	c_1, c_2	$t/t_{0.1}$								
		0.7	0.9	1	1.5	2	2.5	3	3.5	4
0.75	1	1.6729	1.7009	1.7250	1.9464	2.1269	2.6595	2.8211	3.2913	3.7615
0.75	2	1.4771	1.5097	1.5191	1.6734	1.8681	2.1749	2.3709	2.7660	3.1612
0.75	3	1.3853	1.4085	1.4189	1.5417	1.7369	1.9385	2.1494	2.5040	2.8617
0.75	4	1.3275	1.3475	1.3582	1.4622	1.5911	1.7959	2.0108	2.3459	2.6811
0.75	5	1.2886	1.3045	1.3168	1.4081	1.5473	1.6967	2.0332	2.2332	2.5523
0.75	6	1.2602	1.2747	1.2851	1.3683	1.4672	1.7007	1.9479	2.1487	2.4557
0.75	7	1.2393	1.2547	1.2621	1.3378	1.4472	1.6355	1.7830	2.0802	2.3797
0.75	8	1.2226	1.2360	1.2435	1.3134	1.4302	1.5831	1.8235	2.0266	2.3161
0.75	9	1.2078	1.2204	1.2282	1.2923	1.3820	1.5396	1.7782	1.9819	2.2650
0.75	10	1.1963	1.2074	1.2153	1.2754	1.3743	1.5030	1.7394	1.9467	2.2247
0.9	1	1.8203	1.8630	1.8888	2.1080	2.3148	2.6586	3.1914	3.7233	4.2552
0.9	2	1.5880	1.6208	1.6411	1.7733	1.9762	2.3351	2.6099	3.0449	3.4799
0.9	3	1.4731	1.4999	1.5145	1.6468	1.8136	2.0634	2.3261	2.7138	3.1015
0.9	4	1.4029	1.4278	1.4391	1.5451	1.7143	1.9882	2.2769	2.5143	2.8702
0.9	5	1.3573	1.3756	1.3884	1.4766	1.6477	1.8623	2.1401	2.4967	2.7147
0.9	6	1.3230	1.3345	1.3513	1.4449	1.5966	1.7702	2.0409	2.3788	2.7186
0.9	7	1.2956	1.3145	1.3234	1.4046	1.5234	1.7556	1.9626	2.2872	2.6139
0.9	8	1.2739	1.2910	1.3011	1.3727	1.4967	16929	1.8995	2.2161	2.5327
0.9	9	1.2565	1.2709	1.2829	1.3467	1.4745	1.6413	1.8476	2.1551	2.4630
0.9	10	1.2428	1.2571	1.2667	1.3362	1.4557	1.5976	1.8624	2.1036	2.4041
0.95	1	1.9101	1.9557	1.9850	2.1772	2.4655	2.8936	3.4724	3.7221	4.2538
0.95	2	1.6509	1.6843	1.7099	1.8597	2.0704	2.3351	2.8040	3.2713	3.7386
0.95	3	1.5247	1.5511	1.5723	1.7081	1.8813	2.1711	2.4761	2.8888	3.3014
0.95	4	1.4484	1.4721	1.4906	1.5945	1.7681	1.9888	2.2769	2.6564	3.0359
0.95	5	1.3959	1.4191	1.4309	1.5375	1.6908	1.9335	2.2348	2.4967	2.8534
0.95	6	1.3580	1.3837	1.3918	1.4794	1.6344	1.8334	2.1232	2.3810	2.7212
0.95	7	1.3282	1.3465	1.3586	1.4496	1.5910	1.8085	2.0378	2.3761	2.7155
0.95	8	1.3055	1.3217	1.3322	1.4130	1.5564	1.7418	1.9683	2.2948	2.6226
0.95	9	1.2863	1.3035	1.3137	1.3832	1.5281	1.6867	1.9686	2.2294	2.5479
0.95	10	1.2687	1.2847	1.2955	1.3690	1.4806	1.6799	1.9171	2.1728	2.4832
0.99	1	2.0674	2.1200	2.1446	2.3541	2.7057	3.0848	3.7018	4.0482	4.6266
0.99	2	1.7640	1.8000	1.8277	2.0044	2.2301	2.5907	2.9643	3.4614	3.9559
0.99	3	1.6168	1.6501	1.6660	1.8148	2.0012	2.3493	2.7215	3.0396	3.4738
0.99	4	1.5266	1.5543	1.5704	1.7023	1.9075	2.1424	2.4829	2.8984	3.3124
0.99	5	1.4673	1.4933	1.5065	1.6265	1.8082	2.0578	2.3202	2.7069	3.0936
0.99	6	1.4222	1.4443	1.4604	1.5573	1.7359	1.9455	2.2698	2.5663	2.9330
0.99	7	1.3871	1.4092	1.4220	1.5174	1.6805	1.9041	2.1702	2.4578	2.8082
0.99	8	1.3603	1.3816	1.3918	1.4856	1.6364	1.8303	2.0902	2.4383	2.7866
0.99	9	1.3377	1.3574	1.3699	1.4495	1.6004	1.8072	2.0750	2.3609	2.6982
0.99	10	1.3191	1.3372	1.3492	1.4288	1.5704	1.7533	2.0159	2.2961	2.6241

Table 3. OC values based on sampling plan $(n, c_1 = c_2 = 2, t/t_{0.1})$ with provided P^* using ERD.

P*	n	$t/t^0_{0.1}$	$t_{0.1}/t^0_{0.1}$								
			2.75	2.5	2.25	2	1.75	1.5	1.25	1	0.75
0.75	93	0.7	0.9999	0.9998	0.9995	0.9979	0.9911	0.9568	0.7832	0.2471	0.0004
0.75	39	0.9	0.9999	0.9998	0.9992	0.9972	0.9884	0.9467	0.7552	0.2344	0.0010
0.75	27	1	0.9999	0.9997	0.9992	0.9969	0.9874	0.9436	0.7507	0.2437	0.0018
0.75	9	1.5	0.9997	0.9990	0.9971	0.9904	0.9660	0.8782	0.6139	0.1841	0.0079
0.75	5	2	0.9987	0.9965	0.9901	0.9714	0.9166	0.7710	0.4820	0.1693	0.0364
0.75	4	2.5	0.9934	0.9842	0.9615	0.9068	0.7868	0.5730	0.3121	0.1305	0.0654
0.9	123	0.7	0.9999	0.9996	0.9988	0.9955	0.9810	0.9138	0.6326	0.0969	0.0000
0.9	51	0.9	0.9998	0.9995	0.9984	0.9941	0.9762	0.8978	0.6009	0.0936	0.0000
0.9	36	1	0.9998	0.9994	0.9981	0.9931	0.9728	0.8874	0.5819	0.0917	0.0000
0.9	11	1.5	0.9994	0.9983	0.9949	0.9834	0.9434	0.8108	0.4746	0.0880	0.0013
0.9	6	2	0.9977	0.9941	0.9838	0.9543	0.8724	0.6754	0.3483	0.0851	0.0115
0.9	5	2.5	0.9878	0.9714	0.9325	0.8449	0.6720	0.4128	0.1693	0.0492	0.0189
0.95	143	0.7	0.9998	0.9994	0.9981	0.9931	0.9717	0.8774	0.5315	0.0483	0.0000
0.95	59	0.9	0.9997	0.9992	0.9975	0.9911	0.9651	0.8575	0.5000	0.0474	0.0000
0.95	42	1	0.9997	0.9991	0.9970	0.9894	0.9595	0.8408	0.4732	0.0442	0.0000
0.95	13	1.5	0.9990	0.9973	0.9918	0.9741	0.9150	0.7346	0.3521	0.0392	0.0002
0.95	7	2	0.9965	0.9909	0.9756	0.9329	0.8208	0.5777	0.2415	0.0402	0.0034
0.95	5	2.5	0.9878	0.9714	0.9325	0.8449	0.6720	0.4128	0.1693	0.0492	0.0189
0.99	185	0.7	0.9996	0.9988	0.9961	0.9860	0.9454	0.7857	0.3439	0.0097	0.0000
0.99	76	0.9	0.9995	0.9984	0.9949	0.9823	0.9340	0.7569	0.3154	0.0098	0.0000
0.99	54	1	0.9993	0.9981	0.9939	0.9791	0.9242	0.7327	0.2907	0.0090	0.0000
0.99	17	1.5	0.9979	0.9942	0.9830	0.9483	0.8427	0.5734	0.1752	0.0066	0.0000
0.99	9	2	0.9929	0.9820	0.9532	0.8782	0.7034	0.3972	0.1050	0.0079	0.0002
0.99	7	2.5	0.9701	0.9329	0.8519	0.6931	0.4439	0.1835	0.0402	0.0054	0.0012

Table 4. Minimum sample sizes required to prove that the 10^{th} percentile exceeds the values provided, $t^0_{0.1}$, with the probability P^* and different acceptance numbers c_1, c_2 using the bivariate Poisson approximation.

P*	c_1, c_2	$t/t^0_{0.1}$								
		0.7	0.9	1	1.5	2	2.5	3	3.5	4
0.75	1,2	73	30	21	7	4	3	3	3	3
0.75	2,3	108	45	32	10	6	4	4	4	4
0.75	3,4	143	59	42	13	7	5	5	5	5
0.9	1,2	100	41	29	9	5	4	4	3	3
0.9	2,3	140	58	41	13	7	5	5	5	5
0.9	3,4	179	74	52	16	9	7	6	6	6
0.95	1,2	118	65	46	14	8	6	5	5	5
0.95	2,3	161	67	47	15	8	6	5	5	5
0.95	3,4	203	84	59	18	10	8	7	6	6
0.99	1,2	158	65	46	14	8	6	5	5	5
0.99	2,3	207	85	60	19	10	8	7	7	6
0.99	3,4	253	104	73	23	12	9	8	8	8

Table 5. Lot acceptance ratio $d_{0.1}$ for the ERD with a 0.05 producer's risk.

P^*	c_1, c_2	$t/t_{0.1}$								
		0.7	0.9	1	1.5	2	2.5	3	3.5	4
0.75	1,2	1.6277	1.6584	1.6759	1.8583	2.1055	2.4103	2.8882	3.3787	3.8614
0.75	2,3	1.4556	1.4847	1.5047	1.6269	1.8558	2.0362	2.4430	2.8511	3.2584
0.75	3,4	1.3709	1.3923	1.4099	1.5109	1.6532	1.8382	2.2058	2.5735	2.9411
0.9	1,2	1.7643	1.7995	1.8250	1.9933	2.2478	2.6339	3.1600	3.3744	3.8565
0.9	2,3	1.5578	1.5890	1.6093	1.7552	1.9468	2.1870	2.6245	3.0652	3.5031
0.9	3,4	1.4560	1.4799	1.4948	1.6073	1.7979	2.0665	2.3531	2.7453	3.1375
0.95	1,2	1.8401	1.8842	1.9023	2.1054	2.3664	2.8124	3.1607	3.6867	4.2134
0.95	2,3	1.6162	1.6523	1.6702	1.8283	2.0289	2.3161	2.6244	3.0619	3.4993
0.95	3,4	1.5046	1.5310	1.5470	1.6640	1.8552	2.1580	2.4766	2.7453	3.1375
0.99	1,2	1.9808	2.0257	2.0622	2.2535	2.5737	2.9642	3.3749	3.9364	4.4988
0.99	2,3	1.7237	1.7582	1.7822	1.9547	2.1665	2.5366	2.9203	3.4070	3.7058
0.99	3,4	1.5926	1.6227	1.6383	1.7862	1.9659	2.2474	2.5969	3.0297	3.4625

Table 6. OC values based on sampling plan $(n, c_1 = 1, c_2 = 2, t/t_{0.1})$ with provided P^* using ERD.

P*	n	$t/t^0_{0.1}$	$t_{0.1}/t^0_{0.1}$								
			2.75	2.5	2.25	2	1.75	1.5	1.25	1	0.75
0.75	73	0.7	0.9991	0.9981	0.9956	0.9892	0.9705	0.9108	0.7136	0.2444	0.0014
0.75	30	0.9	0.9989	0.9977	0.9947	0.9872	0.9658	0.9001	0.6958	0.2447	0.0030
0.75	21	1	0.9987	0.9974	0.9942	0.9860	0.9630	0.8940	0.6865	0.2466	0.0045
0.75	7	1.5	0.9969	0.9937	0.9865	0.9693	0.9260	0.8154	0.5626	0.1943	0.0147
0.75	4	2	0.9916	0.9837	0.9671	0.9310	0.8526	0.6931	0.4317	0.1672	0.0447
0.75	3	2.5	0.9780	0.9596	0.9243	0.8571	0.7363	0.5481	0.3274	0.1622	0.0942
0.9	100	0.7	0.9983	0.9964	0.9919	0.9801	0.9466	0.8449	0.5540	0.0971	0.0000
0.9	41	0.9	0.9979	0.9957	0.9903	0.9766	0.9387	0.8281	0.5321	0.0982	0.0001
0.9	29	1	0.9976	0.9951	0.9891	0.9739	0.9325	0.8150	0.5140	0.0957	0.0002
0.9	9	1.5	0.9949	0.9897	0.9781	0.9507	0.8838	0.7239	0.4115	0.0871	0.0023
0.9	5	2	0.9870	0.9750	0.9499	0.8967	0.7861	0.5797	0.2943	0.0787	0.0132
0.9	4	2.5	0.9619	0.9310	0.8734	0.7694	0.5990	0.3731	0.1672	0.0577	0.0257
0.95	118	0.7	0.9976	0.9950	0.9888	0.9726	0.9275	0.7955	0.4552	0.0494	0.0000
0.95	49	0.9	0.9970	0.9938	0.9863	0.9671	0.9151	0.7697	0.4240	0.0472	0.0000
0.95	34	1	0.9968	0.9933	0.9851	0.9647	0.9099	0.7605	0.4168	0.0499	0.0000
0.95	11	1.5	0.9924	0.9848	0.9679	0.9286	0.8356	0.6294	0.2883	0.0363	0.0003
0.95	6	2	0.9815	0.9646	0.9298	0.8578	0.7150	0.4729	0.1923	0.0349	0.0036
0.95	5	2.5	0.9422	0.8967	0.8147	0.6753	0.4694	0.2396	0.0787	0.0185	0.0063
0.99	158	0.7	0.9958	0.9911	0.9802	0.9525	0.8777	0.6779	0.2761	0.0097	0.0000
0.99	65	0.9	0.9948	0.9893	0.9763	0.9441	0.8600	0.6468	0.2522	0.0096	0.0000
0.99	46	1	0.9941	0.9879	0.9734	0.9378	0.8467	0.6240	0.2347	0.0092	0.0000
0.99	14	1.5	0.9879	0.9758	0.9494	0.8897	0.7560	0.4933	0.1585	0.0088	0.0000
0.99	8	2	0.9680	0.9393	0.8822	0.7704	0.5712	0.2957	0.0745	0.0060	0.0002
0.99	6	2.5	0.9192	0.8578	0.7512	0.5812	0.3565	0.1469	0.0349	0.0055	0.0014

Table 7. OC values based on sampling plan $(n, c_1 = 2, c_2 = 3, t/t_{0.1})$ with provided P^* using ERD.

P*	n	$t/t^0_{0.1}$	$t_{0.1}/t^0_{0.1}$								
			2.75	2.5	2.25	2	1.75	1.5	1.25	1	0.75
0.75	108	0.7	1.0000	0.9999	0.9996	0.9983	0.9927	0.9630	0.8003	0.2464	0.0002
0.75	45	0.9	0.9999	0.9998	0.9994	0.9978	0.9906	0.9547	0.7758	0.2373	0.0005
0.75	32	1	0.9999	0.9998	0.9993	0.9973	0.9889	0.9486	0.7583	0.2289	0.0008
0.75	10	1.5	0.9998	0.9993	0.9979	0.9930	0.9743	0.9022	0.6594	0.2077	0.0081
0.75	6	2	0.9988	0.9968	0.9911	0.9736	0.9205	0.7731	0.4691	0.1464	0.0252
0.75	4	2.5	0.9965	0.9913	0.9779	0.9435	0.8597	0.6863	0.4301	0.2104	0.1167
0.9	140	0.7	0.9999	0.9997	0.9991	0.9965	0.9850	0.9279	0.6626	0.0985	0.0000
0.9	58	0.9	0.9999	0.9996	0.9988	0.9954	0.9810	0.9140	0.6314	0.0954	0.0000
0.9	41	1	0.9998	0.9995	0.9985	0.9946	0.9782	0.9044	0.6113	0.0927	0.0000
0.9	13	1.5	0.9995	0.9986	0.9956	0.9854	0.9488	0.8202	0.4731	0.0748	0.0006
0.9	7	2	0.9981	0.9951	0.9863	0.9603	0.8845	0.6904	0.3499	0.0766	0.0083
0.9	5	2.5	0.9934	0.9839	0.9601	0.9015	0.7704	0.5356	0.2614	0.0913	0.0393
0.95	161	0.7	0.9999	0.9996	0.9986	0.9948	0.9780	0.8983	0.5683	0.0500	0.0000
0.95	67	0.9	0.9998	0.9994	0.9981	0.9931	0.9720	0.8783	0.5297	0.0469	0.0000
0.95	47	1	0.9998	0.9993	0.9978	0.9921	0.9686	0.8677	0.5130	0.0472	0.0000
0.95	15	1.5	0.9992	0.9978	0.9934	0.9785	0.9265	0.7558	0.3624	0.0348	0.0001
0.95	8	2	0.9973	0.9928	0.9803	0.9439	0.8425	0.6047	0.2515	0.0379	0.0025
0.95	6	2.5	0.9889	0.9736	0.9362	0.8488	0.6699	0.3961	0.1464	0.0357	0.0118
0.99	207	0.7	0.9997	0.9991	0.9971	0.9895	0.9572	0.8181	0.3774	0.0096	0.0000
0.99	85	0.9	0.9996	0.9989	0.9963	0.9867	0.9478	0.7918	0.3471	0.0097	0.0000
0.99	60	1	0.9995	0.9986	0.9956	0.9845	0.9407	0.7725	0.3254	0.0094	0.0000
0.99	19	1.5	0.9985	0.9957	0.9873	0.9597	0.8697	0.6155	0.1941	0.0065	0.0000
0.99	10	2	0.9949	0.9867	0.9642	0.9022	0.7451	0.4394	0.1184	0.0082	0.0002
0.99	8	2.5	0.9757	0.9439	0.8713	0.7200	0.4666	0.1893	0.0379	0.0043	0.0008

Table 8. OC values based on sampling plan $(n, c_1 = 3, c_2 = 4, t/t_{0.1})$ with provided P^* using ERD.

P*	n	$t/t^0_{0.1}$	$t_{0.1}/t^0_{0.1}$								
			2.75	2.5	2.25	2	1.75	1.5	1.25	1	0.75
0.75	143	0.7	1.0000	1.0000	1.0000	0.9997	0.9981	0.9841	0.8575	0.2465	0.0000
0.75	59	0.9	1.0000	1.0000	0.9999	0.9996	0.9974	0.9800	0.8391	0.2439	0.0001
0.75	42	1	1.0000	1.0000	0.9999	0.9995	0.9968	0.9763	0.8221	0.2332	0.0002
0.75	13	1.5	1.0000	0.9999	0.9997	0.9983	0.9908	0.9467	0.7296	0.2159	0.0045
0.75	7	2	0.9999	0.9996	0.9985	0.9935	0.9715	0.8821	0.6135	0.2197	0.0390
0.75	5	2.5	0.9994	0.9981	0.9934	0.9772	0.9242	0.7796	0.5123	0.2497	0.1334
0.9	179	0.7	1.0000	1.0000	0.9999	0.9994	0.9957	0.9661	0.7434	0.0995	0.0000
0.9	74	0.9	1.0000	1.0000	0.9998	0.9991	0.9942	0.9576	0.7134	0.0969	0.0000
0.9	52	1	1.0000	1.0000	0.9998	0.9989	0.9932	0.9522	0.6969	0.0967	0.0000
0.9	16	1.5	1.0000	0.9998	0.9993	0.9964	0.9810	0.8994	0.5738	0.0889	0.0004
0.9	9	2	0.9997	0.9990	0.9961	0.9842	0.9360	0.7675	0.3937	0.0724	0.0052
0.9	7	2.5	0.9979	0.9935	0.9788	0.9322	0.8023	0.5344	0.2197	0.0554	0.0179
0.95	203	0.7	1.0000	1.0000	0.9998	0.9990	0.9932	0.9491	0.6565	0.0498	0.0000
0.95	84	0.9	1.0000	0.9999	0.9997	0.9986	0.9909	0.9366	0.6200	0.0479	0.0000
0.95	59	1	1.0000	0.9999	0.9997	0.9983	0.9893	0.9289	0.6009	0.0479	0.0000
0.95	18	1.5	0.9999	0.9997	0.9989	0.9945	0.9718	0.8588	0.4707	0.0455	0.0000
0.95	10	2	0.9996	0.9985	0.9943	0.9774	0.9116	0.7003	0.3004	0.0385	0.0017
0.95	8	2.5	0.9967	0.9895	0.9668	0.8985	0.7240	0.4149	0.1298	0.0228	0.0056
0.99	253	0.7	1.0000	0.9999	0.9996	0.9977	0.9853	0.8999	0.4717	0.0098	0.0000
0.99	104	0.9	1.0000	0.9999	0.9994	0.9969	0.9809	0.8797	0.4352	0.0099	0.0000
0.99	73	1	1.0000	0.9999	0.9993	0.9962	0.9778	0.8666	0.4144	0.0099	0.0000
0.99	23	1.5	0.9998	0.9993	0.9972	0.9869	0.9378	0.7324	0.2549	0.0069	0.0000
0.99	12	2	0.9992	0.9971	0.9892	0.9587	0.8504	0.5582	0.1609	0.0096	0.0002
0.99	9	2.5	0.9949	0.9842	0.9514	0.8577	0.6396	0.3100	0.0724	0.0088	0.0016

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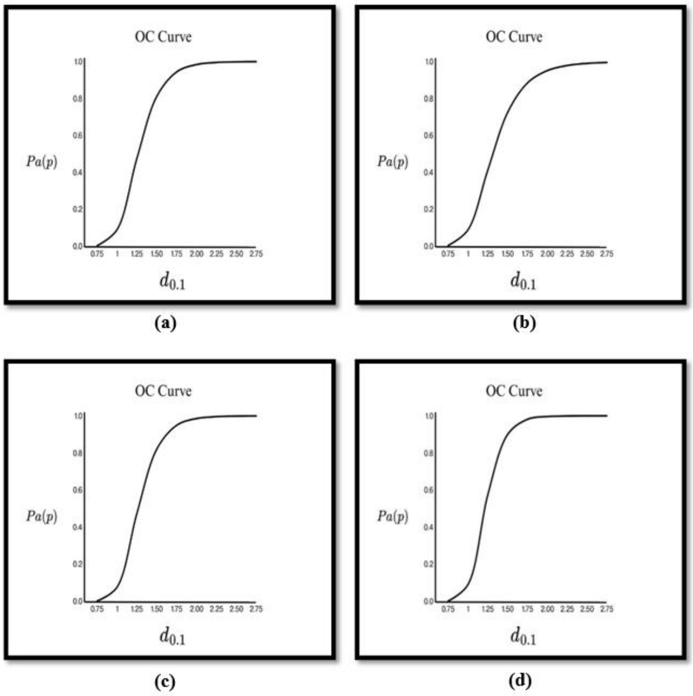

Figure 1. (a) OC curve for the sampling plan (n, c_1, c_2, tt_q^0) = (11,2,2,1.5). (b) OC curve for the sampling plan (n, c_1, c_2, tt_q^0) = (9,1,2,1.5). (c) OC curve for the sampling plan (n, c_1, c_2, tt_q^0) = (13,2,3,1.5). (d) OC curve for the sampling plan (n, c_1, c_2, tt_q^0) = (16,3,4,1.5).

6. Conclusion

The acceptance sampling plans discussed in this article are based on the Exponentiated Rayleigh distribution where the life test is terminated at a predetermined time. The method for building the suggested sampling plans using the bivariate Poisson approximation to calculate the percentiles of the ERD is given. The suggested sampling plans are created by computing the operational characteristic value and minimum sample size that fulfil the risks associated with the producer and customer, respectively. For the ERD, the actual 10^{th} percentile life for the accepted lot also resulted in the values of $c_1, c_2, t/t_{0.1}^0$ and P^*. Use of the acceptance sampling plans that depend on percentiles is necessary to guarantee if their quality of the items exceeds a particular one on the basis of life percentile. A few helpful tables are included to help create an acceptability sampling plan using a one-shot syringe as an example.

Acknowledgement

The authors would express their gratitude to both the anonymous referees for their valuable comments on an earlier version of the paper.

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